Interference Management in Wireless Networks

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Outline

- Interference Alignment
  - degrees-of-freedom
  - channel state issues, ergodic interference alignment
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- Topological Interference Alignment
  - low-rank matrix factorization
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  - finite SNR
  - efficient algorithms
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  - finite SNR
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- Simulation results
  - cellular networks: comparison to frequency re-use
  - ad hoc networks: comparison to graph coloring
  - extensions to MIMO
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- Simulation results
  - cellular networks: comparison to frequency re-use
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  - extensions to MIMO
- Conclusion
As we all know, wireless communication systems are characterized by

1. broadcast during transmission
2. interference during reception
3. random fading
4. path-loss
5. mobility and time-varying channel conditions
6. time-varying traffic patterns
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5. mobility and time-varying channel conditions
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All have been successfully exploited in practical systems (perhaps) with the exception of *interference*. 
Interference Channels

\[ y_i = h_{ii}x_i + \sum_{j \neq i} h_{ij}x_j + z_j, \quad i = 1 \ldots, n \]

- capacity is, by and large, unknown
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Focus, instead, on degrees-of-freedom:

\[ \text{DoF} = \lim_{\text{SNR} \to \infty} \frac{C_{\text{sum}}(\text{SNR})}{\log \text{SNR}}. \]
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- considerably simplifies the analysis
- can lead to physical insight

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- may not "well reflect" actual performance at practical SNRs
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Interference Alignment (Cadambe and Jafar, 2008)

Assume the channel coefficients change over time:

\[ y_i(t) = h_{ii}(t)x_i(t) + \sum_{j \neq i} h_{ij}(t)x_j(t) + z_i(t) \]

Consider \( T \) channel uses:

\[
\begin{bmatrix}
    y_i(1) \\
    \vdots \\
    y_i(T)
\end{bmatrix} = 
\begin{bmatrix}
    h_{ii}(1) \\
    \vdots \\
    h_{ii}(T)
\end{bmatrix}
\begin{bmatrix}
    x_i(1) \\
    \vdots \\
    x_i(T)
\end{bmatrix} + 
\begin{bmatrix}
    h_{ij}(1) \\
    \vdots \\
    h_{ij}(T)
\end{bmatrix}
\begin{bmatrix}
    x_j(1) \\
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Let us assume each transmitter \( j \) sends \( m \) information symbols \( S_j \) across the \( T \) channel uses:

\[ X_j = V_j S_j, \]

where \( V_j \in \mathbb{C}^{T \times m} \) represents the precoding matrix.
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where \( V_j \in \mathbb{C}^{T \times m} \) represents the precoding matrix. Note that the \( i \)-th interference term \( \sum_{j \neq i} H_{ij}V_jS_j \) lives in the range space of the matrix

\[
\left[\begin{array}{cccc}
H_{i1}V_1 & \ldots & H_{i,i-1}V_{i-1} & H_{i,i+1}V_{i+1} & \ldots & H_{in}V_n
\end{array}\right]_{T \times (n-1)m}.
\]
Interference Alignment (Cadambe and Jafar, 2008)

If we can find precoding matrices $V_i \in \mathbb{C}^{T \times m}$ and decoding matrices $U_i \in \mathbb{C}^{m \times T}$ such that

1. $\text{rank}(U_i H_{ii} V_i) = m$

2. $U_i \begin{bmatrix} H_{i1} V_1 & \ldots & H_{i,i-1} V_{i-1} & H_{i,i+1} V_{i+1} & \ldots & H_{in} V_n \end{bmatrix} = 0$

for all $i = 1, \ldots, n$, then each user can send $m$ symbols interference free across $T$ channel uses! (Thus, $\text{DoF} = m$.)
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When $T = n$, $m = 1$ is trivially achieved by time sharing. ($\text{DoF} = 1.$)
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Cadambe and Jafar’s argument relies heavily on the fact that the \( H_{ij} \) are diagonal. They give explicit constructions for the precoding matrices when \( T = O(2^n) \).
Remarks

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Ergodic Interference Alignment (Nazer et al, 2009)

Assuming the $H_{ij}$ vary in an ergodic fashion and that their distributions are symmetric, one can achieve $\text{DoF} = \frac{n}{2}$ without non-causal CSIT:

1. At time $t = 1$ each transmitter $i$ knows all the current channel coefficients $H_{kl}(1)$ and transmits the signal $x_i(1)$.

2. At some future time $t$, we will encounter channel coefficients such that $H_{kl}(t) = -H_{kl}(1)$, for all $k \neq l$.

3. At such a time $t$, each transmitter $i$ transmits the signal $x_i(t) = x_i(1)$.

4. Each receiver $i$ adds its received signals $y_i(1)$ and $y_i(t)$ and thereby perfectly eliminates the interference.

5. Thus each symbol is transmitted interference-free over two channel uses and $\text{DoF} = \frac{n}{2}$ is achieved!

This is not practical, either. (To put it mildly...) Nonetheless, there is a growing literature on attempting to do interference alignment with more reasonable CSIT assumptions. (The jury is still out on what the gains are.)
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Topological Interference Management (Jafar, 2013)

Exploit IA principles under realistic assumptions on CSIT
Knowledge of only the interference pattern at the transmitters
Tight connection to the index coding problem [Birk & Kol'98]

Example:

\begin{align*}
 & t_1 & t_2 & t_3 & t_4 & t_5 \\
 & r_1 & r_2 & r_3 & r_4 & r_5 \\
\end{align*}

(a) Interference pattern

\begin{align*}
 & 2 & 6 & 6 & 6 & 6 \\
 & 6 & 4 & 1 & \leftrightarrow & 00 \\
 & \leftrightarrow & 100 & \leftrightarrow & 0 & \leftrightarrow & 1 \\
 & 0 & \leftrightarrow & 1 & \leftrightarrow & 0 \\
 & 0 & \leftrightarrow & \leftrightarrow & 10 & \leftrightarrow & 0 \\
 & \leftrightarrow & 0 & \leftrightarrow & \leftrightarrow & 1 \\
 & 3 & 7 & 7 & 7 & 7 \\
 & 7 & 5 & & & \\
\end{align*}

(b) Matrix entry pattern
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Example:

(a) Interference pattern

(b) Matrix entry pattern
Note that the following sets of nodes can transmit interference-free:
\{1, 2\}, \{3, 4\}, \{5\}.
For example, \{1, 2\} can transmit in the first time slot, \{3, 4\} in the second, and \{5\} in the third. Thus, 
\[ \text{DoF} = 1 + 3 = 4. \]
Interference Avoidance (Graph Coloring)

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For example, \( \{1, 2\} \) can transmit in the first time slot, \( \{3, 4\} \) in the second, and \( \{5\} \) in the third. Thus, \( \text{DoF} = \frac{1}{3} \). Note that

\[
\begin{bmatrix}
1 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
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= \begin{bmatrix}
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\end{bmatrix}.
\]
Let each transmitter transmit one signal over two channel uses each:
\[ X_1 = \begin{bmatrix} s_1 & 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0 & s_2 \end{bmatrix}, \quad X_3 = \begin{bmatrix} -s_3 & s_3 \end{bmatrix}, \quad X_4 = \begin{bmatrix} -s_4 & s_4 \end{bmatrix}, \quad X_5 = \begin{bmatrix} s_5 & 0 \end{bmatrix}. \]

\[ Y_1, Y_3 \text{ and } Y_5 \] therefore are
\[ Y_1 = \begin{bmatrix} s_1 & 0 \end{bmatrix} h_{11} + \begin{bmatrix} -s_3 & s_3 \end{bmatrix} h_{13} + \begin{bmatrix} -s_4 & s_4 \end{bmatrix} h_{14} + Z_1 \]
\[ Y_3 = \begin{bmatrix} -s_3 & s_3 \end{bmatrix} h_{33} + \begin{bmatrix} s_1 & 0 \end{bmatrix} h_{31} + \begin{bmatrix} s_5 & 0 \end{bmatrix} h_{35} + Z_3 \]
\[ Y_5 = \begin{bmatrix} s_5 & 0 \end{bmatrix} h_{55} + \begin{bmatrix} 0 & s_2 \end{bmatrix} h_{52} + Z_5. \]
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\[ Y_3 = \begin{bmatrix} -s_3 \\ s_3 \end{bmatrix} h_{33} + \begin{bmatrix} s_1 \\ 0 \end{bmatrix} h_{31} + \begin{bmatrix} s_5 \\ 0 \end{bmatrix} h_{35} + Z_3 \]

\[ Y_5 = \begin{bmatrix} s_5 \\ 0 \end{bmatrix} h_{55} + \begin{bmatrix} 0 \\ s_2 \end{bmatrix} h_{52} + Z_5 \]

Note that \( \begin{bmatrix} 1 & 1 \end{bmatrix} Y_1 \), \( \begin{bmatrix} 0 & 1 \end{bmatrix} Y_3 \) and \( \begin{bmatrix} 1 & 0 \end{bmatrix} Y_5 \) are interference-free (Similarly, for \( Y_2 \) and \( Y_4 \)).
Topological Interference Alignment

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Y_1 = \begin{bmatrix} s_1 \\ 0 \end{bmatrix} h_{11} + \begin{bmatrix} -s_3 \\ s_3 \end{bmatrix} h_{13} + \begin{bmatrix} -s_4 \\ s_4 \end{bmatrix} h_{14} + Z_1
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Note that \([1 \ 1]\) \(Y_1\), \([0 \ 1]\) \(Y_3\) and \([1 \ 0]\) \(Y_5\) are interference-free. (Similarly, for \(Y_2\) and \(Y_4\)). Thus, \(\text{DoF} = \frac{1}{2}\).
Topological Interference Alignment

Note that

\[
\begin{pmatrix}
1 & 1 \\
1 & 1 \\
0 & 1 \\
0 & 1 \\
1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
-1 \\
-1 \\
1 \\
\end{pmatrix}
= 
\begin{pmatrix}
1 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 \\
1 & 0 \\
\end{pmatrix}
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\[
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\end{bmatrix}
\]
Key Concept

$S$: set of all pairs $(i, j)$ such that receiver $i$ has interference from transmitter $j$

$$A_{ij} = \begin{cases} 
1 & \text{if } i = j, \\
0 & \text{if } (i, j) \in S \& i \neq j, \\
\times & \text{otherwise}.
\end{cases}$$
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Suppose we have a rank $r$ completion $A = UV$
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Over $r$ time slots:

- transmitter $i$ transmits $v_is_i$, where $v_i$ is the $i$-th column of $V$
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  where $u_i$ is the $i$-th row of $U$
Connection to Low Rank Matrix Completion

\[ \text{DoF} = \frac{1}{r} \]
Connection to Low Rank Matrix Completion

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Challenges:
Connection to Low Rank Matrix Completion

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Challenges:

- What is the minimum possible \( r \) for a given interference pattern?
Connection to Low Rank Matrix Completion

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- For a given \( r \), how to find such matrices (if they exist)?
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Low Rank Matrix Completion Problem:
Connection to Low Rank Matrix Completion

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Low Rank Matrix Completion Problem:

\[
\text{minimize} \quad \text{rank}(A) \\
\text{subject to} \quad A_S = I
\]
Connection to Low Rank Matrix Completion

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Low Rank Matrix Completion Problem:

\[
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\text{minimize} & \quad \text{rank}(A) \\
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\end{align*}
\]

Literature:
- Lots of attention in compressed-sensing and machine learning communities [Fazel, Recht, Parrilo, Candes, Montanari, Sanghavi, Oymak-Hassibi, etc.]
Alternating Projection Method

Instead of searching for the optimal $r$, seek a completion for a fixed $r$:

Matrix Completion Problem: find $A$ subject to $A S = I$

$\text{rank}(A) = r$

The matrix $A$ should lie in the sets:

(S1) Rank $r$ matrices
(S2) Matrices with the entry pattern $\ldots$

Observation: It is very easy to project any given matrix onto the sets (S1) and (S2) individually.
Alternating Projection Method

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(S2) Matrices with the entry pattern $[.]_S = I$

Observation: It is very easy to project any given matrix onto the sets (S1) and (S2) individually
Algorithm 1 Proposed Algorithm: Alternating Projection Method

Let $A^0$ be a random matrix. From $i = 0$ until convergence:

- Project $A^i$ onto (S1): $B^i = \text{svd}(A^i, r)$
- Project $B^i$ onto (S2): $A^{i+1} = [B^i]_{S_c} + I$

Descent method:

- $B^{i+1}$ is the best rank $r$ approximation of $[B^i]_{S_c} + I$

\[
\|B^{i+1} - ([B^i]_{S_c} + I)\|_F^2 \leq \|B^i - ([B^i]_{S_c} + I)\|_F^2
\]

\[
\Rightarrow \|B^i_{S_c} + B^{i+1} - B^i_{S_c}\|_F^2 + \|B^{i+1} - I\|_F^2 \leq \|B^i - I\|_F^2
\]

Convergence to fixed points:

\[
B = \text{svd}(B_{S_c} + I, r)
\]
Algorithm 2 AltMin

Inputs: $n, r, S, P_t$. Initialization: $U_0 \in \mathbb{R}^{n \times r}$ random.

From $i = 0$ until convergence,

- Solve for $V_i$:

  $$\text{minimize} \quad \| (U_{i-1} V_i^T - I)_S \|$$

- Solve for $U_i$:

  $$\text{minimize} \quad \| (U_i V_i^T - I)_S \|$$

If algorithm converges to $V_N$ and $U_N$, output $V_N$ and $U_N$.

$S$ includes the set of indices where $A_{ij} = 0$ and the diagonal.
Numerical Experiments

$$M = \begin{bmatrix}
1 & -2.09 & 0 & 0 & 0.81 \\
-0.47 & 1 & 0 & 0 & -0.39 \\
0 & 1.73 & 1 & 0.69 & 0 \\
0 & 2.52 & 1.45 & 1 & 0 \\
1.23 & 0 & 1.49 & 1.03 & 1 \\
\end{bmatrix} = \begin{bmatrix}
0.93 & 0.89 \\
-0.44 & -0.42 \\
-1.00 & 0.17 \\
-1.46 & 0.25 \\
-0.35 & 1.35 \\
\end{bmatrix}^T \begin{bmatrix}
0.26 & 0.96 \\
-1.80 & -0.47 \\
-0.84 & 0.89 \\
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- Alternating Projection method recovers the optimal rank for all the index coding examples in [Birk & Kol’98] and all the TIM problems in [Jafar’13]

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However, we know from extensive simulations (on much larger problems) that the method does not always yield the optimal rank.
Numerical Experiments

- Alternating Projection method recovers the optimal rank for all the index coding examples in [Birk & Kol’98] and all the TIM problems in [Jafar’13]
- However, we know from extensive simulations (on much larger problems) that the method does not always yield the optimal rank—convergence analysis is still on-going
Towards Practical Wireless Interference Networks

The Alternating Projection method constitutes an efficient way to compute (or lower bound) the DoF of wireless interference networks – provides an opportunity to apply premises of IA under realistic assumptions on CSIT.

Challenges:
- How do DoF results translate to practical SNR?
- How is the capacity affected when you consider geometrically-placed transmitters and receivers, path-loss models, fading and put back in the real channel coefficients?
- How does TIM compare to the baseline, i.e., interference avoidance (frequency reuse, etc)?
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The Alternating Projection method constitutes an efficient way to compute (or lower bound) the DoF of wireless interference networks.
Towards Practical Wireless Interference Networks

The Alternating Projection method constitutes an *efficient* way to compute (or lower bound) the *DoF* of wireless interference networks

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- How does TIM compare to the baseline, i.e., *interference avoidance* (frequency reuse, etc)?
Hexagonal Grid: Setup

- N=8,18,24,32,50 cells.
- 6 users per cell,
- average SNR in each cell = 20db
- average INR from neighboring cell = 12db
- path loss model:
  \[ h_{ij} \sim \mathcal{N}(0, \left(\frac{d_{ij}}{r_0}\right)^{-4.0}) \]

Methods

1. frequency reuse 3 yields \( \text{DoF} = \frac{1}{18} \)
2. with carefully-placed users, and no fading, Jafar exhibits an optimal \( \text{DoF} = \frac{1}{7} \) (257% improvement)
3. we will randomly place 6 users in each cell and will consider fading
Hexagonal Grid: Results

- DoF

<table>
<thead>
<tr>
<th></th>
<th>FreqReuse</th>
<th>Coloring</th>
<th>AltMin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{DoF} )</td>
<td>1/18</td>
<td>1/11</td>
<td>1/9</td>
</tr>
</tbody>
</table>

This is really bad. What is going on?
Hexagonal Grid: Results

- **DoF**

<table>
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- **Sum Rate**

<table>
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<tbody>
<tr>
<td>8</td>
<td>13.5302</td>
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<td>6.2415</td>
</tr>
<tr>
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<td>23.1307</td>
<td>13.0369</td>
</tr>
<tr>
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<td>29.2044</td>
<td>14.9266</td>
</tr>
<tr>
<td>32</td>
<td>41.2803</td>
<td>39.0702</td>
<td>22.3766</td>
</tr>
<tr>
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<td>35.1663</td>
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This is really bad. What is going on?
Let us Look at the Sum Rate

Transmitter $i$ has signal $s_i$, $|s_i|^2 = 1$ and transmits $x_i = v_i s_i \in \mathbb{R}^r$. The power constraint per channel use is $E \|x_i\|^2 = P_t$, which translates to $\|v_i\|^2 = r P_t$.

At receiver $i$, $u_i y_i = u_i v_i h_{ii} s_i + n \sum_{j: A_{ij} \neq 0} u_i v_j \times u_i z_i$.

Therefore the sum rate is $C_{\text{sum}} = \sum_{i=1}^n 1 r \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2 \sigma^2 + \sum_{j: A_{ij} \neq 0} |u_i v_j|^2 \|u_i\|^2 \|v_j\|^2 r P_t \frac{1}{\|h_{ij}\|^2}} \right)$ or $C_{\text{sum}} = \sum_{i=1}^n 1 r \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2 r P_t} \frac{1}{\|h_{ii}\|^2} \right)$.
Let us Look at the Sum Rate

- Transmitter $i$ has signal $s_i$, $E|s_i|^2 = 1$ and transmits $x_i = v_i s_i \in \mathcal{R}^r$. 

The power constraint per channel use is $E \| x_i \|^2 = P_t$, which translates to $\| v_i \|^2 = r P_t$.

At receiver $i$, 

$$u_i y_i = u_i v_i h_{ii} s_i + n \sum_{j: A_{ij} = 0} u_i v_j h_{ij} s_j + u_i z_i.$$ 

Therefore the sum rate is 

$$C_{\text{sum}} = n \sum_{i=1}^1 \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\| u_i \|^2 \| v_i \|^2} \sigma^2 + \sum_{j: A_{ij} = 0} \frac{|u_i v_j|^2}{\| u_i \|^2 \| v_j \|^2} r P_t \| h_{ij} \|^2 \right)$$ 

or 

$$C_{\text{sum}} = n \sum_{i=1}^1 \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\| u_i \|^2 \| v_i \|^2} \frac{1}{r \| v_i \|^2} \sigma^2 + \sum_{j: A_{ij} = 0} \frac{|u_i v_j|^2}{\| u_i \|^2 \| v_j \|^2} r P_t \| h_{ij} \|^2 \right).$$
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- Transmitter $i$ has signal $s_i$, $E|s_i|^2 = 1$ and transmits $x_i = v_is_i \in \mathcal{R}^r$. The power constraint per channel use is $\frac{E\|x_i\|^2}{r} = P_t$, which translates to $\|v_i\|^2 = rP_t$.
- At receiver $i$

$$u_iy_i = u_iv_ih_{ii}s_i + \sum_{j:A_{ij}=0}^{n} u_iv_jh_{ij}s_j + \sum_{j:A_{ij}=\times}^{n} u_iv_jh_{ij}s_j + u_iz_i.$$
Let us Look at the Sum Rate

- Transmitter $i$ has signal $s_i$, $E|s_i|^2 = 1$ and transmits $x_i = v_is_i \in \mathcal{R}^r$.
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\[
u_iy_i = u_iv_ih_is_i + \sum_{j:A_{ij}=0}^n u_i v_j h_js_j + \sum_{j:A_{ij}=\times}^n u_i v_j h Js_j + u_iz_i.
\]

- Therefore the sum rate is

\[
C_{sum} = n \sum_{i=1}^1 \frac{1}{r} \log \left( 1 + \frac{|u_iv_i|^2 |h_is_i|^2}{\sigma^2 \|u_i\|^2 + \sum_{j:A_{ij}=\times}^n |u_i v_j|^2 |h_{ij}|^2} \right)
\]

or

\[
C_{sum} = n \sum_{i=1}^n \frac{1}{r} \log \left( 1 + \frac{|u_iv_i|^2 rP_t |h_{is_i}|^2}{\sigma^2 + \sum_{j:A_{ij}=\times}^n |u_i v_j|^2 |rP_t h_{ij}|^2} \right)
\]
The Sum Rate

\[ C_{\text{sum}} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2 r P_t |h_{ii}|^2} \sigma^2 + \sum_{j:A_{ij}=1}^{n} \frac{|u_i v_j|^2}{\|u_i\|^2 \|v_j\|^2 r P_t |h_{ij}|^2} \right) \]
The Sum Rate

\[
C_{sum} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \frac{rP_t |h_{ii}|^2}{\sigma^2 + \sum_{j:A_{ij}=1} \frac{|u_i v_j|^2}{\|u_i\|^2 \|v_j\|^2} rP_t |h_{ij}|^2} \right)
\]

- Looking at the results of the simulations for "AltMin", the value \(\frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2}\) was often very small.
The Sum Rate

\[ C_{sum} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2\|v_i\|^2} \frac{rP_t |h_{ii}|^2}{\sigma^2 + \sum_{j:A_{ij}=\times} \frac{|u_i v_j|^2}{\|u_i\|^2\|v_j\|^2} \frac{rP_t |h_{ij}|^2}{\|u_i\|^2\|v_j\|^2}} \right) \]

- Looking at the results of the simulations for "AltMin", the value \( \frac{|u_i v_i|^2}{\|u_i\|^2\|v_i\|^2} \) was often very small.
- Therefore we will impose the extra constraint in the algorithm that

\[ \frac{|u_i v_i|^2}{\|u_i\|^2\|v_i\|^2} \geq c, \quad \text{for some } 0 \leq c \leq 1. \]
Algorithm 3 AltMinCon

**Inputs:** $n$, $r$, $S$, $c$, $P_t$. **Initialization:** $U_0 \in \mathcal{R}^{n \times r}$ random.

From $i = 0$ until convergence,

- Solve for $V_i$:
  
  \[
  \begin{align*}
  &\text{minimize} \quad \| (U_{i-1} V_i)_S \| \\
  &\text{subject to} \quad \| v_{ij}^{(i)} \| \leq 1 \text{ and } (u_{ij}^{(i-1)})^T v_{ij}^{(i)} \geq c \| u_{ij}^{(i-1)} \| \quad \forall j
  \end{align*}
  \]

- Solve for $U_i$:
  
  \[
  \begin{align*}
  &\text{minimize} \quad \| (U_i V_i)_S \| \\
  &\text{subject to} \quad \| u_{ij}^{(i)} \| \leq 1 \text{ and } (u_{ij}^{(i)})^T v_{ij}^{(i)} \geq c \| v_{ij}^{(i)} \| \quad \forall j
  \end{align*}
  \]

If algorithm converges to $V_N$ and $U_N$,

- normalize columns of $V_N$ to satisfy transmit power constraint $\| v_{ij}^{(N)} \| \leq \sqrt{r} P_t$.

- output $V_N$ and $U_N$.

$S$ includes only the set of indices where $A_{ij} = 0$. 

Babak Hassibi (Caltech)
AltMin vs AltMinCon

\[ \frac{|\mathbf{u}_j^T \mathbf{v}_j|}{\|\mathbf{u}_j\| \|\mathbf{v}_j\|} \]

\[ j \]

AltMin
AltMinCon

Babak Hassibi (Caltech)
### Hexagonal Grid: Results

<table>
<thead>
<tr>
<th>N</th>
<th>DoF</th>
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Better, but still not quite good enough. What is going on?
Hexagonal Grid: Results

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Better, but still not quite good enough. What is going on?
Back to the Sum Rate

\[ C_{sum} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \frac{rP_t |h_{ii}|^2}{\sigma^2 + \sum_{j:A_{ij}=1} |u_i v_j|^2 \frac{rP_t |h_{ij}|^2}{\|u_i\|^2 \|v_j\|^2}} \right) \]
Back to the Sum Rate

\[ C_{sum} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \frac{r P_t |h_{ii}|^2}{\sigma^2 + \sum_{j:A_{ij}=\times} \frac{|u_i v_j|^2}{\|u_i\|^2 \|v_j\|^2} r P_t |h_{ij}|^2} \right) \]

Simulations show that the interference terms (which are ignored in the structure of \( A \)) may not be very small.
Back to the Sum Rate

$$C_{\text{sum}} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \frac{r P_t |h_{ii}|^2}{\sigma^2 + \sum_{j:A_{ij}=\times} \frac{|u_i v_j|^2}{\|u_i\|^2 \|v_j\|^2}} \right)$$

- Simulations show that the interference terms (which are ignored in the structure of $A$) may not be very small.
- Maximizing $C_{\text{sum}}$ directly is not possible, since we do not know the $h_{ij}$.
Back to the Sum Rate

\[ C_{sum} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} rP_t |h_{ii}|^2}{\sigma^2 + \sum_{j:A_{ij} = \times} \frac{|u_i v_j|^2}{\|u_i\|^2 \|v_j\|^2} rP_t |h_{ij}|^2} \right) \]

- Simulations show that the interference terms (which are ignored in the structure of A) may not be very small.
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Back to the Sum Rate

\[ C_{\text{sum}} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|_2^2 \|v_i\|_2^2} rP_t |h_{ii}|^2 }{\sigma^2 + \sum_{j:A_{ij}=1}^{n} \frac{|u_i v_j|^2}{\|u_i\|_2^2 \|v_j\|_2^2} rP_t |h_{ij}|^2 } \right) \]

- Simulations show that the interference terms (which are ignored in the structure of A) may not be very small.
- Maximizing \( C_{\text{sum}} \) directly is not possible, since we do not know the \( h_{ij} \)—we only want to use topological information.
- However, since we know which cell each user \( j \) is in, from the path-loss model, we have an idea of \( E|h_{ij}|^2 \)
Back to the Sum Rate

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C_{sum} = \sum_{i=1}^{n} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \frac{r P_t |h_{ii}|^2}{\sigma^2 + \sum_{j: A_{ij}=\times} |u_i v_j|^2 \|u_i\|^2 \|v_j\|^2} r P_t |h_{ij}|^2 \right)
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- However, since we know which cell each user \(j\) is in, from the path-loss model, we have an idea of \(E|h_{ij}|^2\).
Proposed Algorithm

We therefore propose

\[
\min_{U \in \mathbb{R}^{n \times r}, V \in \mathbb{R}^{r \times n}} \sum_{(i, j) \in S, i \neq j} |u_i v_j|^2 + \lambda \sum_{(i, j) \in S} |u_i v_j|^2 E_{|h_{ij}|^2}
\]

where \( E_{|h_{ij}|^2} \) depends only on the (distance of the) cells in which receiver \( i \) and transmitter \( j \) live, subject to

\[
|u_i v_i|^2 \|u_i\|^2 \|v_i\|^2 \geq c,
\]

for some \( 0 \leq c \leq 1 \).

The above can also be solved in an alternating minimization fashion.
Proposed Algorithm

We therefore propose

$$\min_{U \in \mathbb{R}^{n \times r}, V \in \mathbb{R}^{r \times n}} \sum_{(i,j) \in S, i \neq j} |u_i v_j|^2 + \lambda \sum_{(i,j) \notin S} |u_i v_j|^2 E|h_{ij}|^2$$

where $E|h_{ij}|^2$ depends only on the (distance of the) cells in which receiver $i$ and transmitter $j$ live, subject to

$$\frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \geq c, \quad \text{for some } 0 \leq c \leq 1.$$
Proposed Algorithm

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### Hexagonal Grid: Results

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<td>1/8</td>
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We get 10%-20% improvement in the sum rate.
### Hexagonal Grid: Results

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We get 10%-20% improvement in the sum rate.
Ad hoc Network Example

- N=100 Tx-Rx pairs randomly placed in a $20 \times 20$ square
- max distance btw Tx-Rx is 1
- average SNR to desired user $= 20db$
- path loss model:
  \[ h_{ij} \sim \mathcal{N}(0, \left( \frac{d_{ij}}{r_0} \right)^{-4.0}) \]

Algorithms

1. greedy Coloring (Coloring)
2. matrix Completion (AltMin)
3. constrained matrix Completion (AltMinCon)
4. rate optimization (RateOpt)
Ad hoc Network Results

- Average values over 25 realizations

We obtain a 40% improvement in the sum rate.
Ad hoc Network Results

- Average values over 25 realizations

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We obtain a 40% improvement in the sum rate.
Ad hoc Network Results

- Average values over 25 realizations

![Graph showing Ad hoc network results]

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We obtain a 40% improvement in the sum rate.
Maximizing the Min-Rate

\[
C_{\text{min}} = \min_{i} \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2 \|P_t h_{ii}\|^2}{\sigma^2 + \sum_{j:A_{ij}=\times} \frac{|u_i v_j|^2 \|P_t h_{ij}\|^2}{\|u_i\|^2 \|v_j\|^2}} \right)
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Maximizing the Min-Rate

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C_{\text{min}} = \min_i \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} \frac{r P_t |h_{i i}|^2}{\sigma^2 + \sum_{j: A_{i j} = 1} |u_i v_j|^2 \|u_i\|^2 \|v_j\|^2} r P_t |h_{i j}|^2 \right)
\]

Here it turns out that, if the \(v_i\) are fixed, maximization over the \(u_i\) is a quasi-convex program, and vice-versa.
Maximizing the Min-Rate

\[
C_{\text{min}} = \min_i \frac{1}{r} \log \left( 1 + \frac{|u_i v_i|^2}{\|u_i\|^2 \|v_i\|^2} r P_t |h_{ii}|^2 \right)
\]

\[
\frac{\sigma^2 + \sum_{j:A_{ij}=\times} |u_i v_j|^2}{r P_t |h_{ij}|^2}
\]

- Here it turns out that, if the \(v_i\) are fixed, maximization over the \(u_i\) is a quasi-convex program, and vice-versa.
- Therefore the min-rate, \(C_{\text{min}}\) can be efficiently maximized using a series of alternating quasi-convex optimizations.
Hexagonal Cell Network

- 64 users (4 x 4 x 4), SNR = 20dB, $\gamma = 3.5$
- Baseline: 3 frequency reuse

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<tbody>
<tr>
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</tr>
<tr>
<td>TIM</td>
<td>0.5x</td>
</tr>
<tr>
<td>Altmin with TIM</td>
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</tr>
<tr>
<td>Altmin with full channel</td>
<td>1.4x</td>
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<th>5%</th>
<th>sum</th>
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<td>-</td>
<td>0.5x</td>
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<tr>
<td>Altmin with TIM</td>
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<td>3x</td>
<td>~0.5x</td>
</tr>
<tr>
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Ad-hoc Network

- 60 users, SNR = 20dB, $\gamma = 3.5$
- Baseline: Greedy coloring

**Sum-rate algorithm**

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**Min rate-algorithm**

<table>
<thead>
<tr>
<th></th>
<th>min</th>
<th>5%</th>
<th>sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIM</td>
<td>-</td>
<td>-</td>
<td>0.75x</td>
</tr>
<tr>
<td>Altmin with TIM</td>
<td>4x</td>
<td>3x</td>
<td>~0.75x</td>
</tr>
<tr>
<td>Altmin with full channel</td>
<td>7x</td>
<td>4x</td>
<td>~0.75x</td>
</tr>
</tbody>
</table>
Discussion and Conclusion

Interference alignment
▶ unreasonable CSIT assumptions (not very practical)

Topological interference alignment
▶ requires only topological information of the network; can significantly improve the DoF
▶ reduces to low rank matrix completion
▶ related to network coding, index coding, secret sharing (when over finite fields)

In practice DoF can be misleading
▶ developed alternative algorithms (moved away somewhat from TIM)
▶ promising preliminary results, especially, for the min-rate: there is something to be had
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3. Can we give conditions for optimality of the solution of AP method, or performance bounds otherwise?

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   - Identify scenarios where we can have an advantage
   - Can we analytically determine the advantage of TIM in ad-hoc and cellular networks using random geometric graph theory?
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- How to combine this with MIMO
- Study of the finite field problem
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- Let each base station have $M$ transmit antennas. Let each user have $Q$ receive antennas (most often $Q = 1$).
- $m_n^k$: the message for user $n$ in cell $k$.
- Each user receives a signal message (above) over $r$ channel uses.
- The transmitted signal from base station $k$ during $r$ consecutive channel uses:

$$S^k = \sum_{n=1}^{N} V_n^k m_n^k,$$

where $V \in C^{r \times M}$ are the linear dispersion matrices.
Signal received at $i$-th user in cell $k$, over $r$ channel uses:

$$Y_i^k = V_i^k h_i^k m_i^k + \sum_{n \neq i} V_n^k h_i^k m_n^k + \sum_{l \neq k} \sum_{n=1}^N V_n^l h_i^l m_n^l + Z_i^k,$$

where $Y_i^k$, $h_i^k$, $h_i^l$, $Z_i^k \in \mathbb{C}^{r \times Q}$. 
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where $Y_i^k, h_i^k, h_i^l, Z_i^k \in \mathbb{C}^{r \times Q}$.

- User $i$ in cell $k$ will have a decoder matrix $U_i^k \in \mathbb{C}^{Q \times r}$ so that it will decode its message as

\[
\text{trace}(U_i^k Y_i^k) = \text{trace}(U_i^k V_i^k h_i^k) m_i^k + \sum_{n \neq i} \text{trace}(U_i^k V_n^k h_i^k) m_n^k + \\
\sum_{l \neq k} \sum_{n=1}^N \text{trace}(U_i^k V_n^l h_i^l) m_n^l + \text{trace}(U_i^k Z_i^k).
\]
The rate to the $i$-th user in cell $k$ is therefore:

$$R_i^k = \frac{1}{r} \log \frac{|\text{trace}(U_i^k V_i^k h_i^k)|^2}{\sigma^2 \|U_i^k\|_F^2 + \sum_{n \neq i} |\text{trace}(U_i^k V_n^k h_i^k)|^2 + \sum_{l \neq k} \sum_{n=1}^N |\text{trace}(U_i^k V_n^l h_l^i)|^2}$$
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- As before we can design the precoding and decoding matrices $V_i^k$ and $U_i^k$ to either maximize the sum rate

$$R = \sum_{i, k} R_i^k,$$

or the minimum rate

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Things are a bit more complicated now because we have matrices and because the channels couple in more tightly. But, otherwise, everything else is the same.
Hexagonal-cell-network: 4 x 4, 3 antennas

- FDMA
- GC
- TIM

MIMO